

Estimation of the minimum required strength of sill mat considering crushing failure induced by wall convergence



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ABSTRACT

Underhand cut-and-fill mining is applied in underground operations where rock masses are either weak or subject to rockburst risks under high stress conditions. In such cases, miners operate beneath a cemented backfill roof or sill mat. Maintaining its stability is crucial for the safety of mining workers and equipment. Existing analytical approaches for evaluating sill mat's stability and strength requirements generally assume rigid rock walls. Nevertheless, recent numerical simulations and field observations demonstrate that pronounced wall convergence induced by the excavation of underlying stopes can trigger crushing failure within the sill mat. This discrepancy suggests that the influence of wall convergence has not been adequately incorporated into current design methodologies. To address this limitation, the present study conducts a series of numerical investigations to reveal the crushing failure mechanism of sill mats subjected to rock wall convergence. Furthermore, an analytical solution is developed to estimate the minimum required cohesion ($c_{\min}$) of sill mat associated with this failure mode. The proposed solution is established without reliance on empirical correction factors, and its validity is verified through comparison with numerical results.

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1 Introduction

Cemented backfill composed of tailings or waste rock is widely applied in underground mining to enhance slope stability, reduce ore loss and dilution, and limit surface disposal of wastes [1–3]. Despite these advantages, its design remains challenging. Economically, binder consumption should be minimized, whereas stability requirements demand sufficient strength. Thus, the design objective is to determine the minimum binder content that ensures stability of side- or base-exposed backfill, corresponding to the minimum required strength.

The required strength can be evaluated using simulation or analytical methods. Numerical modeling can incorporate complex geometries, stress conditions, and material properties, but it requires specialized tools and expertise [4]. In contrast, analytical solutions are more efficient and practical for engineering applications.

For side-exposed backfill, commonly used in open stoping with delayed filling, analytical solutions are well developed, largely derived from [5] and later improvements by Li and co-workers [3, 6–10]. For base-exposed backfill (i.e., sill mats), Mitchell [11] proposed failure modes including caving, sliding, rotation, and flexure. Subsequent studies refined these models, such as the graphical method by Stone [12] and updates by Pakalnis et al. [13]. However, limitations have been identified. For example, Caceres [14] noted an error in the rotational formulation, while Oulbacha [15] showed that rotational failure may be unlikely in steep stopes and that flexural failure is significant only for large-span, thin sill mats. An improved flexural model considering tensile failure at the base center was also proposed.

Compared with the strength models of side-exposed backfill, Mitchell's solution for sill mats [11] has seen limited application. This is partly due to undefined parameters, such as rock–backfill interface strength and the assumption of rigid rock walls. In practice, excavation beneath sill mats induces wall convergence, imposing lateral compression on the backfill [2]. Field observations, such as those from the Lucky Friday Mine, confirm significant wall convergence [16–18]. Excessive convergence may lead to crushing failure, as

also indicated by numerical studies [19–21]. However, existing numerical approaches are site-specific, and an analytical solution for estimating the minimum required strength of sill mat accounting for crushing failure is still lacking.

To address this gap, an analytical solution is developed to estimate the minimum required cohesion ($c_{\min}$) of sill mat associated with crushing failure. The proposed solution is established without reliance on empirical correction factors, and its validity is verified through comparison with numerical results.

2 Numerical investigation of failure modes of sill mats under wall convergence

2.1 Numerical model

Figure 1 presents the numerical model of two stopes. In the figure, H is the stope height; B is the stope width; h_s is the thickness of sill mat. For simplification, the surrounding rock mass is assumed to behave elastically, whereas the sill mat follows a Mohr–Coulomb criterion. The base is fully constrained in all directions. Along the lateral boundaries, vertical displacement is permitted while horizontal displacement is restricted. Plane strain conditions are imposed by fixing all out-of-plane displacements. A uniform surcharge, representing the overburden associated with mining depth (D) and

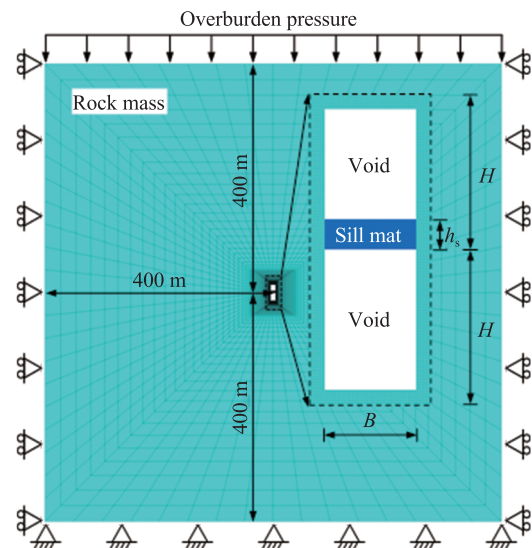


Figure 1 Numerical model after excavation beneath sill mat

model extent, is applied at the top boundary to replace the overlying strata.

Figure 2 illustrates the excavation-and-filling sequence adopted in the simulation. In Step 1, *in-situ* stress field of rock mass is established, with displacements reset to zero after equilibrium to eliminate initial deformation. Step 2 simulates excavation of the upper stope prior to backfilling. In Step 3, a sill mat is constructed using cemented backfill. To reflect delayed filling in practice, deformation induced during the previous excavation stage is excluded by resetting displacements before backfill placement. Step 4 represents underlying stope excavation. At this point, sill mat becomes exposed at its base and is subjected to horizontal compression resulting from wall convergence.

Table 1 summarizes parameters used to investigate sill mat failure under wall convergence at a depth of 400 m. Here, E_r and μ_r denote the Young's modulus and Poisson's ratio of the rock mass and orebody, while E_s and μ_s correspond to those of the sill mat. ϕ_s is the friction angle of sill mat. K_r represents the lateral stress coefficient. The unit weights of the rock mass (γ_r) and sill mat (γ_s) are taken as 27 and 18 kN/m³, respectively. Although not explicitly listed, the tensile strength σ_t of the sill mat is required in the model. Following

experimental observations [22], it is assumed to be 10% of the uniaxial compressive strength (UCS), expressed as Equation (1):

$$\sigma_t = \frac{c_s}{5} \tan \left(45^\circ + \frac{\phi_s}{2} \right) \quad (1)$$

where c_s is the cohesion of sill mat.

2.2 Failure mode and c_{min} of sill mat

Figure 3 illustrates the evolution of yielding pattern within the sill mat in response to variations in its cohesion following excavation of the underlying stope. As observed, localized shear bands initially develop at the basal corners when c_s equals to 160 kPa. With a reduction in cohesion to 148 kPa, these bands propagate along diagonal trajectories and eventually traverse the entire structure. This failure pattern is identified as a crushing mode, which is characteristic of brittle materials subjected to unconfined compression. At this point, the structural integrity of the sill mat is effectively lost.

Accordingly, the sill mat is considered unstable once the shear bands either intersect or fully penetrate the structure. The former condition is typically associated with thicker sill mats [20], whereas the latter is more prevalent in cases where the thickness-to-span ratio is

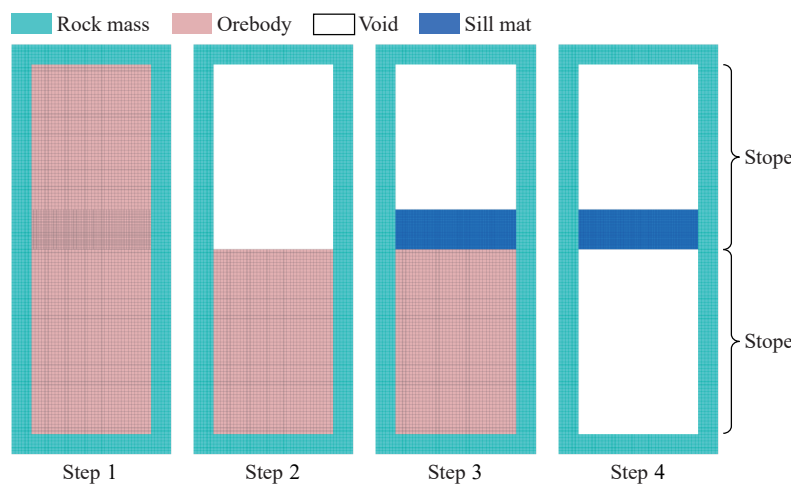


Figure 2 Excavation and backfilling sequence in numerical modeling

Table 1 Numerous parameters used in simulations

Stope geometries			Rock mass/orebody			Sill mat		Lateral pressure coefficient	
H (m)	B (m)	h_s (m)	E_r (GPa)	μ_r	E_s (MPa)	μ_s	ϕ_s (°)	K_r	
14	9	3	20	0.2	200	0.3	30	1.0	

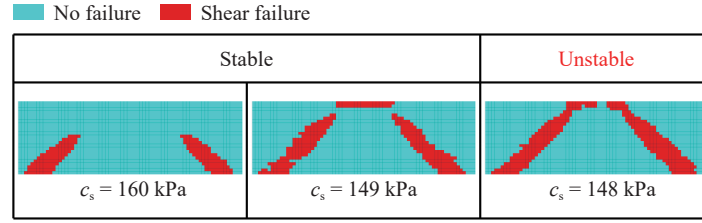


Figure 3 Variation of the yield state of sill mat as a function of c_s

relatively small. For design purposes, the $c_{\min}$ should exceed the critical value corresponding to instability. Based on the analysis, $c_{\min}$ is determined to be 149 kPa. The subsequent section presents a theoretical expression for the $c_{\min}$ of the sill mat, explicitly accounting for the crushing failure mechanism.

3 Analytical solution to determine the $c_{\min}$ of sill mat with crushing failure

Figure 4 provides a schematic representation of sill mat crushing due to wall convergence. For analytical convenience, crushing failure is assumed to occur once the compressive stress σ_s exceeds its UCS, which is determined by the cohesion and friction angle of sill mat material. Under this assumption, the critical state of sill mat can be expressed as Equation (2):

$$\sigma_s = \frac{2c_{\min} \cos \phi_s}{1 - \sin \phi_s} \quad (2)$$

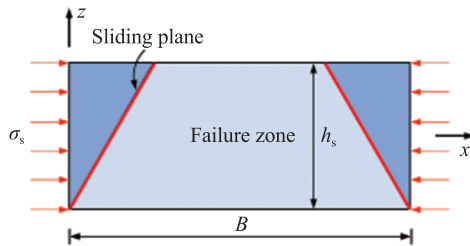


Figure 4 Mechanical model of sill mat with a crushing failure induced by wall convergence

Thus, the $c_{\min}$ of sill mat can be obtained as Equation (3):

$$c_{\min} = \frac{\sigma_s (1 - \sin \phi_s)}{2 \cos \phi_s} \quad (3)$$

where the σ_s in the sill mat generated by wall convergence associated with underlying slope excavation can be obtained by Equation (4) [23]:

$$\sigma_s = \frac{\bar{d}_{12}}{\frac{B(1-\mu_r^2)}{2E_s} + \frac{2\zeta(1-\mu_r^2)}{\pi E_r}} \quad (4)$$

where

$$\bar{d}_{12} = \frac{2K_r \gamma_r D(1-\mu_r^2)}{E_r h_s} \left[\frac{H^2}{2} \arcsin\left(\frac{h_s}{H}\right) - \frac{H^2}{8} \arcsin\left(\frac{2h_s}{H} - 1\right) + \frac{h_s}{2} \sqrt{H^2 - h_s^2} - \frac{2h_s - H}{4} \sqrt{Hh_s - h_s^2} - \frac{\pi H^2}{16} \right] \quad (5)$$

$$\zeta = \frac{H^2}{h_s} \ln\left(\frac{H}{H+h_s}\right) + h_s \ln\left(1 + \frac{H}{h_s}\right) + H + h_s \quad (6)$$

Accordingly, Equations (3)–(6) establish the analytical framework for evaluating the $c_{\min}$ of the sill mat under crushing failure.

4 Verification of the analytical solution using numerical modeling

A set of simulations was conducted to assess the validity of the proposed solution. As summarized in Table 2, the simulation scheme covers a range of geometric and mechanical conditions designed for model verification. Since the analytical formulation does not rely on calibration factors or empirical correction terms, the analytical results can then be interpreted as direct predictions rather than fitted values.

Figure 5 presents the variation in the $c_{\min}$ of the sill mat as influenced by different controlling factors, as determined from both numerical simulations and the proposed analytical solution. A strong agreement between the two sets of results is observed, confirming the reliability of the analytical formulation. This consistency demonstrates that the proposed solution can

Table 2 Simulation scheme covering various parameters for validation of the analytical formulation (D ranging from 400 to 800 m)

Stope geometries			Rock modulus	Backfill modulus	Friction angle of backfill
H (m)	B (m)	h_s (m)	E_r (GPa)	E_s (MPa)	ϕ_s (°)
6–18	9	3	30	300	30
14	6–12	3	30	300	30
14	9	2–6	30	300	30
14	9	3	10–50	300	30
14	9	3	30	150–600	30
14	9	3	30	300	25–40
14	9	3	30	300	30

accurately predict sill mat strength without the need for empirical calibration.

As illustrated in Figure 5a, for a given mining depth, the $c_{\min}$ increases with increasing H . This trend can be attributed to the enhanced convergence of the surrounding rock induced by excavation beneath the sill mat, which generates greater horizontal compressive deformation and stress within the sill mat. Consequently, a higher strength is required to maintain stability.

Figure 5b shows that, at a constant mining depth, the $c_{\min}$ decreases as B increases from 6 to 12 m. This trend arises from the reduction in compressive stress associated with rock wall convergence as the stope width increases, thereby lowering the strength requirement. In addition, the results suggest that such convergence may contribute to reduced binder consumption, offering potential economic benefits.

As shown in Figure 5c, for a fixed depth, the analytically predicted $c_{\min}$ decreases with increasing h_s from 2 to 6 m. This reduction is associated with the enhanced load-bearing capacity of a thicker sill mat, which alleviates rock wall convergence and reduces the induced compressive stress. However, the numerical results indicate that the $c_{\min}$ approaches a constant value when the thickness exceeds approximately 4 m. This phenomenon can be explained by the stress state in the lower portion of the sill mat, which tends toward uniaxial compression under crushing conditions. Owing to the significant stiffness contrast between the rock mass and the sill mat, as shown in Table 2, further increases in thickness exert a limited influence on wall convergence. As a result, the $c_{\min}$ stabilizes at

greater thicknesses. These observations suggest the existence of an optimal sill mat thickness that balances safety requirements and economic efficiency.

Figure 5d indicates that, for a fixed mining depth, the $c_{\min}$ decreases as E_r increases from 10 to 30 GPa, followed by a more gradual decline as E_r further increases to 50 GPa. This trend reflects the reduction in rock wall convergence associated with increased stiffness of the surrounding rock, which leads to lower compressive stresses acting on the sill mat. For a given value of E_r , the $c_{\min}$ continues to increase approximately linearly with depth. These findings highlight the importance of accounting for rock mass stiffness in sill mat design.

As shown in Figure 5e, for a constant mining depth, the $c_{\min}$ increases markedly as E_s rises from 150 to 600 MPa. This behavior arises because a stiffer sill mat undergoes smaller deformation under rock wall convergence but develops higher internal compressive stresses, thereby requiring greater cohesion to prevent crushing failure. These findings also indicate that a sill mat with relatively lower stiffness may still maintain stability, providing opportunities to reduce binder consumption.

Finally, Figure 5f shows that, for a fixed mining depth, the $c_{\min}$ decreases with increasing ϕ_s from 25° to 40°. This trend is due to the dependence of UCS on both cohesion and friction angle, as described in Equation (2). Accordingly, an increase in ϕ_s reduces the required cohesion for a given strength demand.

5 Discussion

In this study, a representative failure mode of a sill mat subjected to rock wall convergence induced by

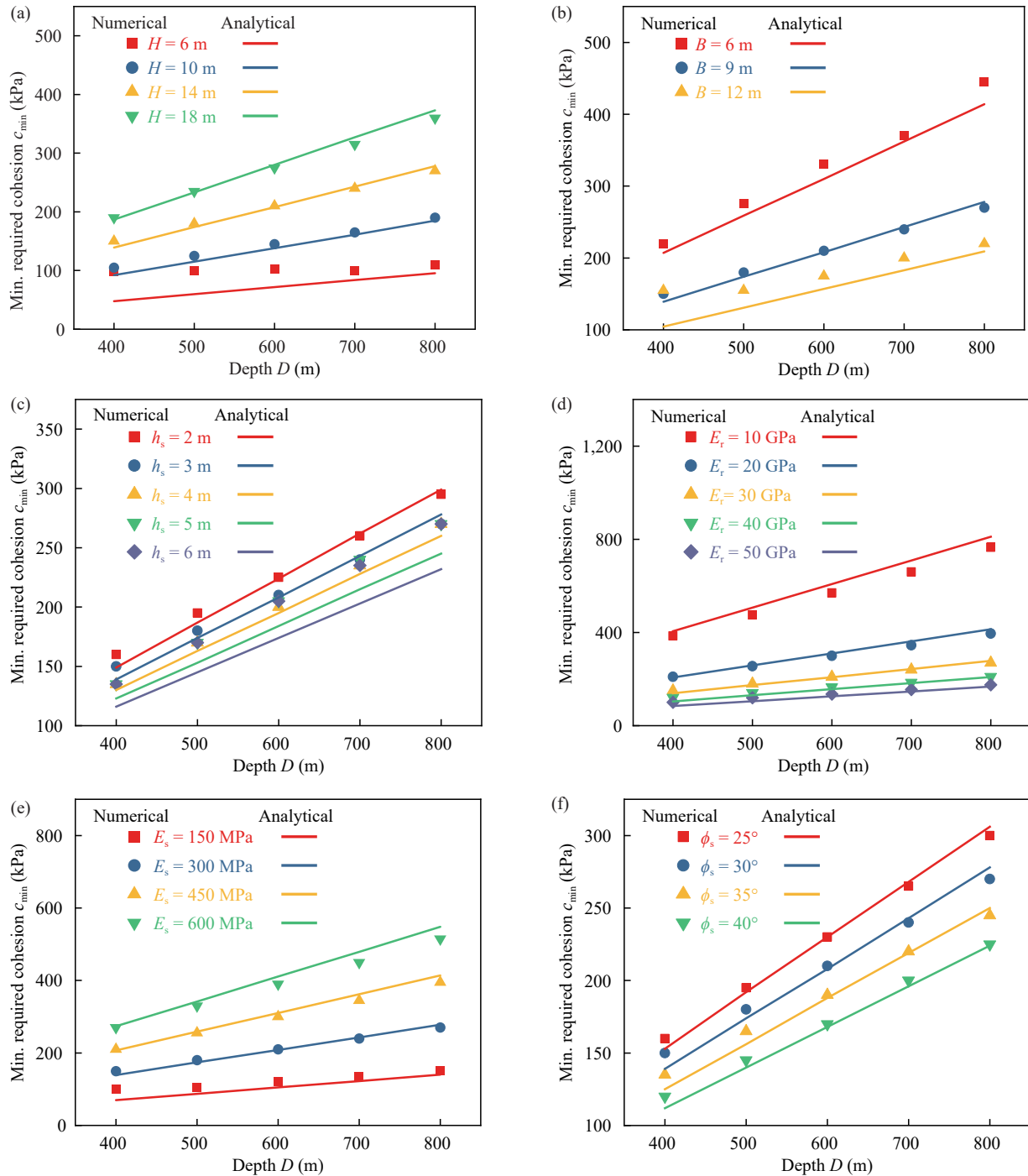


Figure 5 Variation of the $c_{\min}$ of the sill mat as a function of mining depth and key parameters: (a) stope height H , (b) stope width B , (c) sill mat thickness h_s , (d) rock mass modulus E_r , (e) backfill modulus E_s , and (f) friction angle ϕ_s , as determined by numerical simulation and the analytical solution

the excavation of the underlying stope was identified through simulations. Based on this mechanism, an analytical formulation was established to estimate the $c_{\min}$ of the sill mat. The proposed solution is derived without the use of empirical correction or calibration

factors. A close correspondence between numerical results and analytical predictions confirms the validity and predictive capability of the model. Consequently, the formulation provides a practical means for estimating $c_{\min}$. Nevertheless, several simplifying assumptions

were introduced during its development, and the associated limitations are discussed below.

First, the surrounding rock mass was treated as a linearly elastic medium. The suitability of this assumption depends on factors such as rock mass quality, *in-situ* stress conditions, and mining depth. For shallow excavations or competent rock masses, linear elasticity provides a reasonable approximation, as demonstrated in previous studies [24–26]. In contrast, for deep mining conditions or highly fractured and weak rock masses, this assumption becomes less representative, and numerical approaches are generally more appropriate. However, analytical solutions remain attractive in engineering practice due to their simplicity, accessibility, and reduced dependence on specialized expertise [4]. They can be readily implemented using common tools (e.g., spreadsheets), enabling rapid estimation and parametric analyses. To maintain analytical tractability, the assumption of linear elasticity was therefore adopted in both the numerical model and the analytical derivation. Future work should focus on extending the formulation to account for nonlinear rock mass behavior.

Second, the sill mat was idealized as an elastoplastic material governed by the Mohr–Coulomb criterion. Since the objective was to determine the minimum strength requirement, the analysis focused on the stress state prior to failure, under which the sill mat response was assumed to be linear elastic. Accordingly, the elastic solution developed by Fan et al. [23] for evaluating the compressive stress induced by wall convergence was employed. It should be noted, however, that the use of an elastic formulation may lead to an overestimation of compressive stress, which could partly explain discrepancies between analytical and numerical results.

Third, the influence of overlying fill was neglected. In practical conditions, the void above the sill mat is often occupied by uncemented or lightly cemented backfill, which may affect stress redistribution. This aspect was omitted for simplicity in both the numerical simulations and analytical model. Further research is required to incorporate the stress contribution of overlying backfill into the estimation of the $c_{\min}$.

Fourth, the analytical model was developed under plane strain conditions, and is therefore applicable

primarily to stopes with large extents in the out-of-plane direction [20, 21]. Extension to three-dimensional conditions represents an important direction for future investigation.

Fifth, the inclination of the stope walls was not considered, and a vertical geometry was assumed. In practice, stope walls are often inclined to varying degrees, and the effect of stope dip on $c_{\min}$ warrants further study.

Sixth, the self-weight of the sill mat was neglected in the formulation of the crushing failure mechanism. This simplification is justified by the fact that failure is predominantly governed by horizontal compressive stresses induced by wall convergence, while vertical stresses due to self-weight are comparatively minor. The validity of this assumption is supported by the strong agreement between analytical and numerical results.

Finally, the tensile strength of sill mat was assumed to be 10% of its UCS in the numerical simulations. Recent studies [27, 28] suggest that this ratio may be underestimated for cemented backfill. Therefore, the influence of tensile-to-compressive strength ratio on sill mat stability should be further investigated.

6 Conclusions

A crushing failure mechanism of sill mat, induced by rock wall convergence, was identified through numerical simulations. On this basis, an analytical solution was developed to estimate the $c_{\min}$ of sill mat. The proposed formulation is derived without introducing any empirical correction or calibration factors. Its validity and applicability are supported by the close agreement between the numerical results and analytical predictions. Therefore, the proposed solution can be employed to estimate the $c_{\min}$ of sill mats subjected to rock wall convergence caused by excavation of the underlying stope.

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Author contributions

Chuan Fan: Analytical solution, numerical simulation, writing-original draft & editing. **Guangsheng Liu:** Funding acquisition, supervision, methodology, writing-review & editing. **Xiacong Yang:** Supervision, conceptualization, methodology, writing-review & editing. **Lijie Guo:** Supervision, methodology, writing-review & editing. **Li Li:** Conceptualization, supervision, methodology, writing-review & editing.

Declarations

The authors declare that they have no conflict of interests.

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